SENIOR HIGH SCHOOL CURRICULUM GUIDE

MATHEMATICS 10
EFFECTIVE SEPTEMBER, 1963

MATHEMATICS 20
EFFECTIVE SEPTEMBER, 1964

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Senior High School Curriculum Guide

for

Mathematics 10

(Effective September, 1963)

and

Mathematics 20

(Effective September, 1964)

Department of Education

September, 1963

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PREFACE

Mathematics curricula are in the process of revision from first grade to university post-graduate level. Various authorities submit differing statements regarding reasons for this revision. A consensus of these statements is the following:

1. An explosion of knowledge has taken place in the field of mathematics in the last fifty years resulting in new areas of study such as topology and abstract algebra. Courses in high school besides enabling other objectives to be attained must prepare students to advance in various areas of mathematics at the university level. Emphasis is therefore placed on the basic structures and patterns which pervade the entire field of mathematics e.g. mathematical systems and functions. This emphasis on basic structures and patterns is evident in contemporary developments in the curriculum of all subject areas.

2. The conventional grade placement of various topics is being re-assessed in order to determine when concepts can be most effectively and efficiently taught. For example, in some experimental programs much more geometric content has been taught successfully to junior high school students.

3. Curriculum change in the last decade has been characterized by increasing emphasis on the intellectual aims of education.

4. Some mathematical concepts are decreasing in importance: for example, logarithmic solutions of oblique triangles. Because these concepts are of decreasing importance their emphasis in high school mathematics curricula is decreasing.

The subject of Euclidean geometry has been under close scrutiny by curriculum revision groups. A variety of solutions has been put forward with none meeting general acceptance. The Senior High School Mathematics Subcommittee feels it would be unwise to proceed with the general revision of Mathematics 10 until the new junior high school mathematics program is fully defined. By this time experimentation with various approaches to high school geometry will have been carried forward and the conclusions from this experimentation will be available. The subcommittee feels, however, that a revision of the present course using the materials already authorized and supplemented by suggested references, is desirable. In this way the objectives of the Mathematics 10 and Mathematics 20 programs can be more effectively attained. The revised courses are discussed in detail in this curriculum guide.

OBJECTIVES OF THE SENIOR HIGH SCHOOL MATHEMATICS PROGRAM

While the academic, vocational and commercial sequences of courses have different specific objectives the common general purposes of Senior High School Mathematics courses are as follows:-

1. To develop an understanding of mathematics as a creation of man and to develop an appreciation of the contribution of this discipline to the progress of civilization.

2. To develop precision in thought and expression.

3. To develop and maintain an understanding of the operations and concepts of mathematics.

4. To develop and maintain skill in mathematical operations.

5. To develop powers of logical analysis of problems and of presenting their solution in a clear and precise manner.

PERSPECTIVE

The effective mathematics teacher must know how the concepts taught in a particular grade fit into the general development of the subject. In this way needless repetition of the work of previous grades will be avoided. In addition, the teacher will know which concepts are new and require careful presentation and which concepts involve a review of the notions previously taught. If the teacher is aware of concepts to be developed in succeeding grades, problems resulting from carrying an area of study too far can be avoided. The teacher of Mathematics 10 or Mathematics 20 therefore should familiarize himself with the curriculum outline for Mathematics 9, 10, 20, 30, and 31.

To assist the teacher, a chart of the mathematics program in Grade IX and the academic program of the senior high school is presented on the following page. It has been found that staff meetings and meetings at other administrative levels are advantageous in acquainting teachers with the mathematics program of Grades IX to XII. If these meetings are held early in the fall they will be of maximum value.

CONTENT OF MATHEMATICS COURSES

	Mathematics 9	Mathematics 10 (Effective September, 1963)	Mathematics 20 (Effective September, 1964)	Mathematics 30	Mathematics 31
ARITHMETIC	Perimeter: triangle, square, rectangle, parallelogram, circle. Area: triangle, square, rectangle, parallelogram, circle. Volume: cube, rectangular solid, cylinder. Factors: multiples, powers, roots. English v.s. Metric Measurement Common, decimal fractions; terminating and repeating decimals; line, circle and bar graphs. Graphs of formulas; percent: discount, interest, taxes, etc.	Area: sector of circle, lateral surface of cone, surface of sphere. Volume: prism, pyramid, cone, sphere.			
GEOMETRY	Vocabulary: kinds of angles, kinds of triangles, kinds of figures with four or more sides. Also such terms as ratio, congruence, vertical, horizontal, bisect. All terms related to circle measurement. Constructions:- Bisect an angle (two methods). Right bisector of straight line. Distance from point to a line. Complete a triangle given incomplete data. Inductive Proof:- Sum of angles of a triangle. Congruence SSS, SAS, SAA. Angles at base of isosceles triangle. Parallel lines. Theorem of Pythagoras. Properties of similar rectilinear figures.	Vocabulary:- Basic geometric terms. Constructions:- Basic geometric constructions. Formal Methods of Proof Basic Assumptions Definitions, assumptions, theorems, and other important relationships with respect to: congruence, parallelism, area, similarity, Pythagorean relationship, locus, circle.			
TRICONOMETRY			Ratio, Proportion, Variation:- Application of proportion to geometry and trigonometry (sin, cos and tan relations). Variation: Direct and inverse variation defined. Solution of problems by 'k" method, and proportion. Logarithms: Introduced as an application of indices. Use of logarithmic tables to facilitate multiplication, division, finding powers and roots.		Functions of an acute angle. Solution of right triangles. Approximate numbers: logarithms. The slide rule. Solution of right triangle by logarithms. Trigonometric functions of any angle. Radian measure: mil measure. Line values and graphs of functions. Fundamental relations. Identities. Functions of two angles. Oblique triangles. Inverse functions. Complex numbers and hyperbolic functions.
ALGEBRA	Vocabulary:- Literal numbers, monomial, binomial, algebraic expression, literal factor, numerical factor, coefficient, like terms, conditional equations, negative integers, absolute value. Skills and Understandings:- Rules for operation with algebraic expression; Simple linear equations; Use of simple equations in the solution of problems; Changing the subject of a formula; Operations with negative intergers; Operations with algebraic fractions.	Equations with Two Unknowns:- Solutions by elimination and substitution. Graphs: locus, cartesian coordinates. —location of points; —graph of first degree equations by means of a table of values. Constant, variable, function, slope, intercept. Craphing a first-degree equation by the slope-intercept method. Linear graph as an example of locus. Craph of a linear system. Quadratics: Function and equation. Product of two linear factors. Factoring a trinomial. Solving quadratic equations. Basic Operations: Square of binomials and trinomials, cube of binomial, product of sum and difference of two terms. Multiplication of polynomials. Division of polynomial by monomial, binomial or trinomial. Factoring: common factor, difference of squares, trinomial.	Craphs: extension of concepts introduced in Mathematics 10. Factoring: grouping, incomplete square, sum and difference of cubes, factor theorem. Application of Factoring: H.C.F., L.C.M. Addition, subtraction, multiplication, division of fraction, complex fractions. Equations: literal and fractional. Indices: laws; fractional, zero, negative; use in multiplication, division, solving equations, factoring. Rational and Irrational Numbers: definition, entire and mixed radicals. Addition, subtraction, multiplication and division of radicals. Conjugate radicals. Equations involving radicals. Quadratic Equations: square root using tables Definitions for quadratic function and equation. Solution by factoring, completing the square, by formula, graphing. Quadratic Function: graph, significance of a, b, and c in graphing functions of $ax^2 + bx + c$. Maximum and minimum values, vertex, axis of symmetry, turning point, abscissa, ordinate.	Functions Linear Function and Applications Quadratic functions and equations Rational functions of one or more variables. Ratio and proportion. Systems of equations. Series of numbers. Progressions. Permutations and Combinations Mathematical Induction Binomial Theorem	

SOME PRINCIPLES FOR EFFECTIVE MATHEMATICS INSTRUCTION

Mathematics teaching can be made more effective by applying the following principles:-

1. Readiness is an important aspect of every learning situation. It is especially important in the subject of mathematics because of the sequential nature of the material. Before new material is presented a brief review of concepts necessary for the understanding of the material is strongly advised.

2. Teaching is most effective when students are encouraged to formulate their own generalizations. As this approach is time-consuming it cannot be used throughout a course. It should be utilized, however, as much as time permits.

3. The effective mathematics teacher realizes the importance of growth in mathematical reading ability and is aware that the reading of mathematics involves a special vocabulary and special skills. Towards this end the mathematics teacher assumes these responsibilities.
   (a) He teaches the students the special skills that are necessary. e.g. reading slowly, re-reading several times, etc.
   (b) He encourages the reading of related references and library books about mathematics.
   (c) He gives attention to the development and review of a mathematics vocabulary.

4. Some students learn best through the sense of sight, some through the sense of touch. Concrete materials such as diagrams, pictures, charts, and mathematical models should be utilized whenever possible in the development of a mathematical concept.

5. If students are to grow in accordance with their ability some attempt must be made to accommodate children with modest ability and children with much ability. Individualized instruction, varied assignments, drill with selected students, library reading assignments, term papers and problems of the week are some procedures that have been used by successful teachers to differentiate instruction.

MATHEMATICS 10

Objectives of Mathematics 10

1. To review and extend students' knowledge of basic geometric understandings as part of the literacy of today's average citizen.
2. To give high school students an appreciation of the nature of mathematical proof; to distinguish between inductive proof and deductive proof.
3. To give students some appreciation of the development of a logical mathematical system.
4. To review and extend the students' knowledge of formulas for the area and volume of common geometric figures and to give practice in applying these formulas in problem situations.
5. To review basic algebraic notions and to introduce the following new concepts:-
   —graphing linear equations;
   —solution of simultaneous equations in two unknowns;
   —introduction to quadratic equations;
   —elementary types of factoring.

COURSE OUTLINE FOR MATHEMATICS 10

Content

Texts:-

MATHEMATICS FOR CANADIANS—Book 2, Bowers, Miller, Rourke.
LOCUS AND THE CIRCLE — Bowers, Miller, Rourke.

Teacher References:-

FUNCTIONAL MATHEMATICS — Book 4, Dean and Moore.
PLANE GEOMETRY — Welchons, Krickenberger and Pearson.
HIGH SCHOOL GEOMETRY — Keniston and Tully.

Geometry

—basic geometric terms;
—basic assumptions;
—methods of proof;
—basic geometric construction;
—definitions, assumptions, theorems, and other important relationships with respect to the following:-
—congruence;
—parallelism;
—area;
—similarity;
—Pythagorean relationship;
—locus;
—circle;

Arithmetic

Area of rectangle, circle, cube, rectangular solid, cylinder, sector of a circle, triangle, lateral surface of a cone, sphere; volume of a cube, rectangular solid, cylinder, prism, pyramid, cone, sphere.

Algebra

1. Review of Grade IX Algebra:-
   (1) Rules for basic operations involving signed number with applications.
   (2) Equations with one unknown.
   (3) Problem solving.
2. Equations with two unknowns:-
   (1) Algebraic solution by:-
      (a) elimination;
      (b) substitution.
   (2) Problem solving, involving two unknowns.
3. Graphs:-
   (1) Locus—definition and examples.
   (2) Cartesian co-ordinate system—location of points.
   (3) Graph of a first degree equation by means of a table of values.
   (4) Concepts to be developed: constant, variable, function, slope, intercept. Graphing a first degree equation by the slope-intercept method. Linear graph as an example of locus.
   (5) Graph of a linear system.
   (6) Formulas, as extension of function concept.
   (7) Functional notation.
4. An introduction to quadratics:-
   (1) Quadratic function and equation defined.
   (2) Product of two linear factors.
   (3) Factoring a trinomial.
   (4) Solving the quadratic equation.
   (5) Problems involving quadratic equations.
5. Basic operations:-
   (1) Type products:-
      (a) square of binomial;
      (b) square of trinomial;
      (c) cube of binomial;
      (d) product of sum and difference.
   (2) Multiplication of polynomials.
   (3) Division: polynomial by monomial, polynomial by binomial or trinomial.
   (4) Factoring:-
      (a) common factor;
      (b) difference of squares;
      (c) trinomial.
   (5) Simple applications of factoring.
   (6) Problem solving.

Geometry

Objectives:-

1. To review and extend the students' knowledge of basic geometric understandings as part of the literacy of today's average citizen (Everyday Uses of Geometry.)
2. To give high school students an appreciation of the **nature of mathematical proof**; to distinguish between inductive proof and deductive proof.
3. To give students some appreciation of the development of a logical mathematical system.

Re Objective No. 1 — Everyday Knowledge of Geometry:-

In today's world the knowledge of basic arithmetic is not enough. Adults need to know some elementary facts of algebra and geometry as well. However, a knowledge of deductive geometry is not essential since the basic geometric relations can be taught inductively and accepted as plausible conclusions. This is the approach commonly followed in junior high school geometry.

The Grade X pupil has already met many elementary geometric facts and concepts in his junior high school work. Among these are:-

1. Notion of point, line, straight line.
2. Notion of angle and kinds of angles.
3. Adjacent, vertically opposite, supplementary and complementary angles.
4. Ideas of parallelism and of perpendicularity.
5. Polygons, triangles, kinds of triangles, parallelograms, special forms of the parallelogram.
6. Experimental treatment of the angle sum of any triangle.
7. Notion of congruence; three conditions for congruence of triangles.
8. Properties of similar rectilinear figures.
9. Experimental treatment of the Pythagorean Theorem.
10. Notion of converse.
11. Certain geometric constructions such as:-

    —how to bisect a line or angle;

    —how to draw a perpendicular to a line;

    —how to copy a triangle.

These geometric facts and concepts need review, reinforcement and extension. In addition to these concepts the students' knowledge concerning areas and volumes of familiar plane figures and solids needs to be strengthened. Furthermore, students must now become acquainted with some of the elementary properties of the circle and must develop the concept of locus.

Re Objective No. 2 — Nature of Proof

The pupils should learn:-

(a) to distinguish between inductive and deductive reasoning, to appreciate the limitations of the former and its vital role in the development of knowledge; to apply both inductive and deductive reasoning in mathematics;

(b) to recognize the nature and importance of definitions and assumptions in deductive reasoning, to recognize the characteristics of good definitions and to formulate simple definitions of familiar concepts, to be alert to hidden assumptions and to challenge unsubstantiated statements;

(c) to understand the characteristics of indirect proof and to apply it in simple situations, both geometric and non-geometric (mathematical and non-mathematical);

(d) to distinguish between analysis and synthesis as methods of proof; to apply analytic procedures to the solution of original theorems, exercises and constructions, then having discovered the solution, to formulate a synthetic proof.

The abler students or classes of superior pupils should understand the inverse, converse and contrapositive of a propositional statement such as a definition, an assumption or a theorem. They should use the contrapositive in proof and should be able to distinguish between "necessary" and "sufficient" conditions.

Re Objective No. 3: Geometry as an Example of a Logical Mathematical System

The main aim in Grade X Geometry should be to lead pupils to understand, in an elementary way, the nature of a logical mathematical system and how it is developed. Their appreciation at best will be imperfect but a carefully laid introduction now will lead to deeper appreciation in the future. It is probable that algebra may soon provide a more satisfactory means for introducing pupils to the wonders of mathematical systems but for the present geometry must be relied upon to achieve this objective.

The course should be so organized as to bring out clearly the role of each of the four categories of ideas and concepts which comprise any logical system of mathematics, namely:-

—undefined terms;
—definitions;
—assumptions;
—theorems.

The arbitrary nature of each of the above can be illustrated by having pupils compare different elementary geometry texts. (The three primary references would serve very well in this regard).

Texts and References for the Geometry Section
Texts:-

MATHEMATICS FOR CANADIANS — Book 2 (hereafter referred to as Book 2).

LOCUS AND THE CIRCLE.

Primary References:-

PLANE GEOMETRY — Welchons, Krickenberger, Pearson, Ginn and Co., 1958, hereafter referred to as Welchons.

HIGH SCHOOL GEOMETRY—Keniston and Tully, Ginn and Co., 1960, hereafter referred to as Keniston.

FUNCTIONAL MATHEMATICS — Dean and Moore, Holt Rinehart and Winston, 1958, hereafter referred to as Dean and Moore.

It is recommended that each teacher have a copy of Dean and Moore as well as at least one of the other two primary references.

NOTE:-

It is recommended that students use mathematical tables whenever possible. Teachers should encourage students to obtain individual copies of **Knott's Mathematical Tables** (revised edition) or **Castle's Five-figure Logarithmic and Other Tables** and should give instruction in the use of these tables.

Content:-

A. Undefined terms:-

point, line, straight (line), **angle, plane, figure, revolution** or **degree** are often considered to be undefined terms. Relationship such as **on, contain, equal, congruent, greater than,** and **between** are also commonly left undefined.
Students have an intuitive notion of these terms and relationships which is quite satisfactory for the purpose of this course.

B. Definitions:-

By building upon undefined terms, definitions can be obtained. Definitions are statements giving the meaning of the term defined. Though they are arbitrary in nature, conventional mathematics definitions are accepted on a world-wide basis to simplify communication of ideas. It should be pointed out to students that all terms do not have precise definitions, that a term such as "democracy" has a variable definition and that before two people can communicate effectively in a given discussion using this term, they must agree with what they mean by the term "democracy".

Characteristics of a Good Definition:-

It names the term to be defined.

It places the term in the smallest class to which it belongs.

It gives the definite characteristics which distinguishes the term from other members of the class.

It is reversible.

Furthermore, a definition should not contain any terms which have not been either previously defined or accepted as undefined and it should not contain more details than is necessary.

References:-

Book 2, pp. 97 and 158.
Welchons, pp. 9 and 10.
Dean/Moore, p. 8.

Initial Definitions:-

The following terms and concepts may suffice for the introductory phases of the course. The first few have been arranged in chains; this device may help to emphasize to pupils how one definition is often dependent upon a prior one. Undefined terms have been capitalized.

LINE—**line segment, broken line, polygon, triangle;**
RAY—(optional); **distance between two points; equal line segments;**
ANGLE—**straight, right, acute, obtuse, reflex;**
REVOLUTION—degree or
DEGREE—revolution
Adjacent angles, vertically opposite angles, supplementary angles, complementary angles;
Midpoint of a line segment, bisector of an angle;
Circle (and its parts);
Triangle, isosceles, scalene, right-angled.

Other definitions may be introduced as the need arises. Formal definitions need not be formulated for all terms but an intuitive understanding should be assured. **Pupils should be asked to reserve a section of their notebooks for definitions.**

References:-

Book 2, Chapter 4.
Welchons, Chapter 1.
Dean/Moore, Chapter 2.

The list at the end of Chapter 2 of Dean/Moore is considered to be unnecessarily detailed.

C. Assumptions:-

The system of geometric thought must start with statements which are accepted as being true. Obviously at the beginning no previously proved theorems can be used as general statements. Available only are the undefined terms and the definitions which have been accepted or formulated.

The following assumptions are suggested for the geometry section. Not all need be introduced at the beginning though a majority might well be developed at this time. It should be pointed out to students that mathematicians now use the terms assumption, axiom, and postulate synonomously to identify relationships accepted as being true. **Both sets of assumptions may well be placed on a large wall chart for easy reference.**

Set A:-

The assumptions in this set apply to algebra as well as to geometry and are often called axioms. They are found at the beginning of most introductory geometry texts. Students are familiar with a number of these axioms having used them in solving simple algebraic equations.

1. Quantities equal to the same quantity or to equal quantities are equal to each other. (57)
2. Any quantity may be substituted for an equal quantity. (58)
3. If equals are added to equals the sums are equal.
4. If equals are subtracted from equals the remainders are equal. (58)

5. If equals are multiplied by equals the products are equal. (198)
6. If equals are divided by equals the quotients are equal. (198)
7. The whole is equal to the sum of its parts and is greater than any one of them. (141)

The following two are sometimes added:-

Any quantity is equal to itself.

Like powers or like roots of equals are equal.

NOTE:-

It should be made clear to students that the pronoun "equals" may refer to numbers or to magnitudes such as measure of length, area, volume or time.

References:-

Book 2 as indicated by numbers in parentheses after each assumption.
Welchons, pp. 52-54.
Keniston, pp. 57-64.
Dean/Moore, pp. 98-107.
Keniston and Dean/Moore are very good references for this topic.

Set B:-

The following set of assumptions sometimes referred to as "postulates of geometry" are needed before proceeding to the development of geometric proofs.

1. A straight line segment can be extended indefinitely in two directions.
2. One straight line and one only can be drawn through two points.
3. A straight line segment is the shortest distance between two points.
4. Two straight lines cannot intersect in more than one point.
5. A line segment has one and only one midpoint.
6. An angle has one and only one bisector.
7. One circle, and only one can be drawn with a given point as center and any given line segment as radius.
8. Radii of the same circle or of equal circles are equal.
9. If circles have equal radii (or diameters) they are equal.
10. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other. (S-A-S=S-A-S) (This takes the place of proposition 1 in the text).
11. Two triangles are congruent if two angles and a side of one are equal respectively to two angles and a side of the other. (A-S-A=A-S-A).
12. Corresponding parts of congruent triangles are equal.
13. If two adjacent angles together form two right angles, their exterior arms are in the same straight line.

Additional Assumptions:-

1. The two remaining theorems referring to the congruence of triangles **may also be assumed.** They are, the one involving three sides (S-S-S) and the one involving the hypotenuse and one other side of a right triangle. (Cf. Prop. 6 in LOCUS AND THE CIRCLE).

2. Since the teaching of indirect proof may prove unfruitful with pupils of modest ability the teacher may wish to introduce one or two additional assumptions when discussing parallel lines. He may assume the validity of Proposition 12 or of both Proposition 12 and 16. For abler pupils in the class the method of indirect proof should be demonstrated.

3. The other important assumptions to be introduced when appropriate are:-
   (a) Two straight lines which intersect one another cannot both be parallel to the same straight line (Playfair's Axiom).
   (b) The area of a rectangle is equal to the product of the measures of its length and its width times the unit area.

4. Although the Welchons, Keniston, and Dean/Moore texts spell out a number of additional assumptions, all of which are hidden or considered unimportant in Book 2, the teacher may well ignore them.

In comparing references, teachers may note that certain early theorems in Keniston are left as assumptions in Welchons. It is important that pupils gradually come to appreciate that there can be a variation in the number of relationships which are assumed. Some assumptions are made for convenience because they appear plausible and their proof may be involved. As the course proceeds some relationships originally assumed may be proven. A good example illustrating that proofs may vary depending upon those relationships assumed is found in Dean/Moore, pages 127 to 142.

The relationships:-

If a transversal meets two parallel straight lines, the interior alternate angles are equal:

and

If a transversal meets two straight lines making the interior alternate angles equal the two straight lines are parallel:

are proven directly and indirectly on the bases of different assumptions.

References:-

Welchons, pp. 56, 57.
Keniston, pp. 65, 66.
Dean/Moore, pp. 107, 122.

D. Theorems:-

It is suggested that only the key theorems indicated below be formally developed. All the remaining theorems and all the problems should be treated as exercises.

Theorems to be learned and their suggested order of presentation:-

Theorem 1: (Proposition 10)
Theorems 2 and 3: (Proposition 2 and 21)
Theorem 4: (Proposition 11)
Theorem 5: (Proposition 12)
Assumption: Playfair's Axiom
Theorem 6: (Proposition 16)
Theorem 7: (Proposition 19)

Assumption: The area of a rectangle is equal to the product of the measures of its length and its width, times the unit area.

Theorems 8, 9, 10, 11, 12: (Propositions 22, 25, 26, 28, 29)

Theorem 13: (Proposition 4—LOCUS AND THE CIRCLE)

Assumption: (Proposition 6—LOCUS AND THE CIRCLE)

Theorem 14, 15, 16, 17, 18, 19, 20, 21: (Propositions 7, 8, 14, 18, 19, 21, 24, 26—LOCUS AND THE CIRCLE)

Notes Re Development of Theorems:-

Teachers will be aware that the first theorem: IF TWO STRAIGHT LINES INTERSECT THE VERTICALLY OPPOSITE ANGLES ARE EQUAL is Proposition 10 in Book 2 but Theorem 1 in Welchons and in Dean/Moore. Due to its simplicity it is very suitable for introducing classes to the basic elements of formal proof. Before additional theorems are introduced the class should be given a number of simple exercises whose proof rests upon the definitions and assumptions thus far set out or upon Theorem One. Algebraic situations should be included. Excellent sources for suitable exercises are contained in Welchons, Chapter 3 and Keniston, Chapter 2. Attention of teachers who have access to Welchons is drawn particularly to pages 58, 59 and 72, for oral exercises based upon assumptions and definitions, to pages 68-70 and to page 75 for a good test.

The distinction between inductive and deductive reasoning should now be beginning to emerge in the minds of pupils. However, if the distinction is to become clear, repeated attention to the two processes will be required as the course in geometry unfolds.

Exercises involving the congruence assumptions SAS and ASA may now be introduced. As Book 2 contains few suitable exercises, the references may be used as follows:-

Welchons, pp. 84-105.

Keniston, pp. 83-102.

Dean/Moore, pp. 155-160.

A judicious choice from among the numerous exercises duplicated or placed on the board should be made. Many of the simple diagramatic exercises should be treated orally and only a few selected for formal written proof.

At this point the teacher is advised to introduce Theorems 2 and 3 (the isosceles triangle and its converse) and to bring out the concept of converse. The third congruence assumption (SSS) could either be introduced at this time or deferred until a little later. Keniston pages 105-119, which is the best reference for this section, also discusses the significance of the words **in, all, some and no** in mathematical statements, and the danger of reasoning from converses. Teachers desiring more formal exercises than are included in Keniston may choose from Book 2, pages 107-109, 177-178 and from Dean/Moore, pages 167-168.

Up to this point the authorized text may be of very little good use. From now on a teacher may continue to develop his course with little or no use of the text, relying upon systematical development, students' notebooks, and the guidance of one of the recommended references. He may, however, now turn to Book 2 as his basic text and supplement this by limited use of the other references. In either case he will be guided by the course outline.

It would appear appropriate at this time to treat as exercises the six construction problems contained on pages 121-130 of the text, to know the accompanying definitions, and to do a few of the related exercises.

Teachers should not miss the opportunity of providing an interesting and valuable interlude before going on to the next section on parallel lines by assigning to their classes the investigation of the two problems contained on pages 133-135 of the text. These problems illustrate the value of experimental techniques and inductive methods of discovering hypotheses as well as the dangers of inductive methods of proof.

Teachers following Book 2 will now move to Chapter 7 and the following topics:-

Theorem 4 (former Proposition 11) — preceded by exercises 9-13, pp. 140.

Definition of parallel lines.

Characteristics of indirect proof.

Theorem 5 (former Proposition 12).

Theorem 6 (former Proposition 16).

Definitions of Parallelogram and Rectangle.

Exercises should include former Propositions 13, 14, 15, 17, 18 and exercises 7, 8, 9 and 10 on pp. 156 and 157, together with any other exercises from the reference texts. Weak classes may accept as assumptions Theorems 4 and 5.

As an enrichment topic for bright pupils the concepts of the **contrapositive** and the **inverse** of a conditional statement and their relationship to the conditional statement and its converse should be discussed. The notion of necessary and sufficient conditions may also be introduced. Brief discussions and exercises regarding the contrapositive and inverse of a conditional statement are found in Welchons, pp. 150, 153 and 355; in Keniston, pp. 182 and 184; and in LOCUS AND THE CIRCLE, pp. 392-393.

Following Theorem 7 (Proposition 19) the following postulate must be emphasized before proceeding to Theorem 8:-

The area of a rectangle is equal to the product of the measures of its length and its width, times the unit area.

Work through the remaining sections of geometry in Book 2 should proceed without difficulty except that due to the limited time available the content must be considerably abbreviated.

Pages 376-380 of LOCUS AND THE CIRCLE require careful teaching to give students a grasp of the concept of locus.

Theorem 13 (Proposition 4) should be followed by a selection of exercises on pages 385-386. Proposition 6 can be assumed. Following Theorem 14 (Proposition 7) special emphasis may be placed on exercises 6 and 7, page 392. Pages 392-402 contain optional material. Consequently, the concepts omitted should be recognized when making a selection from the review exercises on pages 402-404.

The definitions, assumptions and relationships discussed on pages 405-410 are basic to the work on the circle. Perhaps one or two of the relations in exercises 1-10, pages 409 and 410, should be proven. The remainder may be accepted as being true. Exercises following Theorem

15 (Proposition 8) should include Propositions 9-11 and exercises 11, 13 and 14, page 415. The definitions on page 414 are important.

Proposition 12 may be assumed or treated as an exercise. The same applies to the corollary to Proposition 12, Proposition 13 and its corollary. Before Theorem 16 (Proposition 14) is taught, the definition on page 418 should be carefully noted.

If students are able to prove exercises 2 and 3, page 419, they have in essence proven Theorem 16. Exercises following Theorem 16 should include Propositions 15, 16 and 17, and a selection from the exercises on page 423-424.

The remainder of the geometry content of the LOCUS AND THE CIRCLE may be treated in the same manner as that indicated above.

References—Locus:-

Welchons, pp. 346-379.
Keniston, pp. 285-315.

References—Circle:-

Welchons, pp. 284-345.
Keniston, pp. 211-266.

E. Exercises and Proof:-

A primary goal of geometry teaching should be to develop the student's ability to think mathematically. In applying his thinking to the solution of geometric problems the student should learn to set down his solution in a logical and somewhat formal manner. He must substantiate his statements with reasons.

When teaching the method of formal proof it should be stressed from the outset that all steps should be included unless it is specifically stated that certain steps may be omitted in a given situation. Furthermore all necessary authorities should be quoted. It is unnecessary and unwise to require the student to learn the numbers of given assumptions and theorems. He should be required to quote sufficient of the authority to identify it. Suitable contractions and abbreviations should be developed for the more commonly used authorities.

Teachers should require pupils to do only sufficient exercises that they become familiar with the use of a particular key theorem or group of theorems.

The use of previously proven exercises should be freely admitted as authorities in the solution of subsequent exercises.

Length of the Geometry Section of the Course

As in the past, it is anticipated that teachers will move through the geometry section of the course at varying rates of speed. It is expected, however, that most teachers will complete this section in 24-26 weeks. It should be noted that though the content of the course has been increased by the addition of the LOCUS AND THE CIRCLE the number of theorems to be proven formally has been reduced from 30 to 21.

Arithmetic

Objectives:-

The objectives of this section of the course are:-

1. To recognize the common geometric figures, such as the cone, pyramid, etc.
2. To emphasize the relationship between the various measures of common geometric forms. For example:-

$$\frac{\text{volume of a pyramid}}{\text{volume of a prism}} = \frac{\text{volume of a cone}}{\text{volume of a cylinder}}$$

(if the pyramid and the prism have equal bases and if the cone and the cylinder have equal bases).

3. To develop skill in finding areas and volumes of geometric figures through substitution in appropriate formulas.

The memorization of seldom-used formulas should not be stressed.

Content:-

Book 2, Chapter 14, Areas and Volumes.

It is suggested that the chapter be treated in the following manner:-

1. Brief review of previously learned formulas.
2. Presentation of new formulas with emphasis on their interrelationships.
3. Instruction and practice in substituting in formulas.
4. Selection of exercises from the review set on pages 290-294.

If the above arithmetic is taught following the algebra, then sections 117 and 118, pp. 285-290, of Dean/Moore can be used to provide enrichment material for better classes. In these sections literal equations are discussed and their relation to formulas is developed.

Models may be used to good advantage in teaching students to recognize the various geometric figures.

Algebra

Objectives:-

To review basic algebraic notions and to introduce the following new concepts:-

—graphing linear equations;
—solution of systems of equations in two unknowns;
—introduction to quadratic equations;
—elementary factoring.

Content:-

1. Review of Grade IX Algebra:-
   (1) Rules for basic operations involving signed number with applications.
   (2) Equations with one unknown.
   (3) Problem solving.

References:-

Book 2, pp. 1-19.
Dean/Moore, Sections 100-130, excluding sections 107, 108, 117 and 118, and related exercises.

2. Equations with Two Unknowns: (This section may be taken after Section 3. See note below.)

   (1) Algebraic solution by:-
      (a) elimination;
      (b) substitution.

   (2) Problem solving, involving two unknowns.

References:-

Book 2, pp. 20-31.
Dean/Moore, Sections 154-178, pp. 362-410, excluding sections 160, 162 and related exercises.

NOTE:-

Many authors including Dean/Moore introduce the solution of systems of equations in two unknowns by graphical methods. This approach has the advantage of providing a more meaningful introduction to the mathematical concepts involved. The teacher wishing to follow this procedure will experience no difficulty in proceeding to section 3 — Graphs — before teaching section 2.

3. Graphs:-

   (1) Locus — definition and examples.
   (2) Cartesian co-ordinate system — location of points.
   (3) Graph of a first degree equation by means of a table of values.
   (4) Concepts to be developed: constant, variable, function, slope, intercept. Graphing a first degree equation by the slope-intercept method. Linear graph as an example of locus.
   (5) Graph of a linear system.
   (6) Formulas, as extension of function concept.
   (7) Functional notation.

References:-

Book 2, pp. 31-46.
Dean/Moore, Sections 132-152, pp. 321-358.

4. An Introduction to Quadratics:-

   (1) Quadratic function and equation defined.
   (2) Product of two linear factors.
   (3) Factoring a trinomial.
   (4) Solving the quadratic equation.
   (5) Problems involving quadratic equations.

References:-

Book 2, pp. 349-363.
Dean/Moore, Section 201, 202, 203, pp. 455-462.

5. Basic Operations:-

   (1) Type products:-
      (a) square of binomial;
      (b) square of trinomial;

(c) cube of binomial;
(d) product of sum and difference.

(2) Multiplication of polynomials.

(3) Division: polynomial by monomial, polynomial by binomial or trinomial.

(4) Factoring:-
(a) common factor;
(b) difference of squares;
(c) trinomial.

(5) Simple applications of factoring.

(6) Problem solving.

Reference:-

Book 2, pp. 364-373.

To teach the algebra section of the course effectively, material beyond that in Book 2 is necessary. Those sections indicated from Dean/Moore are useful for this purpose.

MATHEMATICS 20

Objectives of Mathematics 20:- (Effective September, 1964)

1. To review algebraic notions developed in previous courses.
2. To extend the students' understanding of number systems and further develop skill in operating with numbers through a study of radicals, exponents and logarithms.
3. To extend the students' knowledge of factoring.
4. To develop the concepts of ratio, proportion and variation.
5. To extend students' understanding of linear and quadratic functions and equations and graphing procedures related to these.
6. To extend students' ability to apply knowledge of algebraic operations to the solution of problems.

COURSE OUTLINE FOR MATHEMATICS 20

Prerequisite:-

"B" or better standing in Mathematics 10.

Text:-

MATHEMATICS FOR CANADIANS — Book 3, Bowers, Miller, Rourke.

References:-

ALGEBRA — ITS BIG IDEAS AND BASIC SKILLS — Book 2, Aiken, Henderson and Pingry.

ALGEBRA — Book 2 (Modern Edition), Welchons, Krickenberger and Pearson.

FUNCTIONAL MATHEMATICS — Book 4, Dean and Moore.

Content:-

1. Review of Basic Skills:-
   (1) Basic Laws of Algebra:-
      (a) Commutative and Associative Laws of Addition.
      (b) Commutative, Associative and Distributive Laws of Multiplication.
   (2) Basic skills involving signed number: subtraction, addition, substitution, multiplication, division.
   (3) Solution of equations in one and two unknowns.
   (4) Problem solving.
2. Graphs:-
   (1) Locus—definition and examples.
   (2) Cartesian coordinate system.
   (3) Straight line graph introduced as locus. Mechanical drawing, using table of values.
   (4) Definitions of constant, variable, function. Application of these terms to Formulas (Set 4, p. 13, Book 3). Extend function concept to graph of linear equation.

(5) Consistency and solution of linear systems — algebraic and geometric methods.

(6) Extension of intercept method to other graphs, e.g. circle, ellipse.

3. Ratio, Proportion, Variation:-

(1) Ratio and proportion: definitions and application to general problems.

(2) Applications of proportion to:-
   (a) Geometry;
   (b) Trigonometry (sin, cos and tan relations).

(3) Variation: direct and inverse variation defined. Solution of problems by:-
   (a) k method;
   (b) proportion.

Maintain function concept in application to problems.

4. Factoring:-

(1) Factoring by the following methods:-
   (a) common factor;
   (b) trinomial;
   (c) difference of squares;
   (d) grouping;
   (e) incomplete squares;
   (f) sum and difference of cubes;
   (g) factor theorem.

(2) Applications of factoring:-
   (a) H.C.F. and L.C.M.
   (b) multiplication and division of fractions;
   (c) addition and subtraction of fractions;
   (d) use of the multiplier (-1);
   (e) complex fractions.

5. Equations and Problems:-

(1) Literal equations — apply function concept.

(2) Fractional equations.

(3) Problems requiring the use of equations.

6. Indices:-

(1) Basic laws of indices.

(2) Meaning of fractional, zero, and negative indices.

(3) Application of rules of indices to: multiplication, division, solving equations, factoring.

(4) Logarithms:-
   (a) introduced as an application of indices;
   (b) use of logarithmic tables to facilitate:-
      i. multiplication;
      ii. division;
      iii. finding powers and roots.

7. Rational and Irrational Numbers:-
   (1) Review the number system. Define rational number, irrational number.
   (2) Entire and mixed radicals.
   (3) Addition, subtraction, multiplication, and division of radicals.
   (4) Conjugate radicals.
   (5) Equations involving radicals.

8. Quadratic Equations:-
   (1) Square root using tables.
   (2) Definition for:-
      (a) quadratic function;
      (b) quadratic equation.
   (3) Solution of the quadratic equation:-
      (a) by factoring;
      (b) by completing the square;
      (c) by formula;
      (d) graphically.
   (4) The quadratic function:-
      (a) significance of a, b, and c, in the general function $ax^2 + bx + c$, with regard to the sketch of the function;
      (b) discussion of the graph of a quadratic function;
      (c) correct use of language with reference to the graph of a quadratic function (maximum and minimum values, vertex, axis of symmetry, turning point, abscissa, ordinate).
   (5) Problems leading to quadratic equations.

SUGGESTIONS FOR TEACHING MATHEMATICS 20

(Effective September, 1964)

Prerequisite:-

"B" or better standing in Mathematics 10.

Text:-

MATHEMATICS FOR CANADIANS — Book 3, Bowers, Miller, Rourke.

References:-

ALGEBRA — ITS BIG IDEAS AND BASIC SKILLS — Book 2, Aiken, Henderson and Pingry, McGraw-Hill, 1960 (hereinafter referred to as Aiken).

ALGEBRA — Book 2 (Modern Edition), Welchons, Krickenberger and Pearson, Ginn and Company, 1962 (hereinafter referred to as Welchons).

FUNCTIONAL MATHEMATICS — Book 4, Dean/Moore, Holt Reinhart and Winston, 1958 (hereinafter referred to as Dean/Moore).

NOTE:-

It is recommended that students use mathematical tables whenever possible. Teachers should encourage students to obtain individual copies

of **Knott's Mathematical Tables** (revised edition) or **Castle's Five-figure Logarthmic and Other Tables** and should give instruction in the use of these tables.

Content:-

1. Review of Basic Skills:-
   (1) Basic Laws of Algebra:-
      (a) Commutative and Associative Laws of Addition.
      (b) Commutative, Associative, and Distributive Laws of Multiplication.
   (2) Basic skills involving signed number; subtraction, addition, substitution, multiplication, division.
   (3) Solution of equations in one and two unknowns.
   (4) Problem solving.

The basic laws of operation of algebra — commutative law of addition and multiplication, associative law of addition and multiplication and the distributive law — should be reviewed. It should be emphasized that these laws of operation apply to the number systems that are studied, and also apply to algebraic operations since the latter are, in fact, operations with numbers. Other properties of zero and one may be studied at this point if the teacher so desires. The broader the students' understanding of operations in the real number system, the more meaningful algebraic operations become.

The basic algebraic axioms listed below should be reviewed prior to the solution of equations.

1. Quantities equal to the same quantity or to equal quantities are equal to each other.
2. Any quantity may be substituted for an equal quantity.
3. If equals are added to equals the sums are equal.
4. If equals are subtracted from equals the remainders are equal.
5. If equals are multiplied by equals the products are equal.
6. If equals are divided by equals the quotients are equal.
7. The whole is equal to the sum of its parts and is greater than any one of them.
8. Any quantity is equal to itself.
9. Like powers or like roots of equals are equal.

NOTE:-

It should be made clear to students that the pronoun "equals" may refer to numbers or to magnitudes such as measures of length, area, volume or time.

Though the study of linear inequalities as discussed in Welchons, pp. 46-49 is not on the course, this section, as well as the section on compound sentences of the first degree in one variable (pp. 49-52) are considered to be excellent enrichment for superior classes.

2. Graphs:-

(1) Locus — definition and examples.

(2) Cartesian coordinate system.

(3) Straight line graph introduced as locus. Mechanical drawing, using table of values.

(4) Definitions of constant, variable, function. Application of these terms to Formulas (Set 4, p. 13). Extend function concept to graph of linear equations.

(5) Consistency and solution of linear system — algebraic and geometric methods.

(6) Extension of intercept method to other graphs, e.g. circle, ellipse.

Sections 1, 2, 3, 4 and 5, are essentially a review of Grade X material. Dean/Moore, pp. 321-361, is an excellent reference for all topics other than algebraic methods for the solution of linear systems. Book 3 is a reasonably good reference for the latter.

A teacher who wishes to present a more complete development of the function concept and of linear functions may refer to Aiken, pp. 219-259 or Welchons, pp. 127-145.

Teachers should note that Welchons introduces the graph of linear inequalities along with the graph of linear equations. Better classes might well include the graphing of linear inequalities as part of their program.

The extension of the intercept method to graphing the circle, ellipse and hyperbola might well be postponed until after the class has been taught to graph the quadratic function (sec. 8-4 (b)). At this time students will be more familiar with second degree functions.

References:-

Book 3, chpt. 2 and pp. 269-273.
Aiken, pp. 288-294.
Welchons, pp. 344-352.
Dean/Moore, pp. 321-361.

3. Ratio, Proportion, Variation:-

(1) Ratio and proportion: definitions and application to general problems.

(2) Applications of proportion to:-
(a) Geometry;
(b) Trigonometry (sin, cos and tan relations).

(3) Variation: direct and inverse variation defined solution of problems by:-
(a) k method;
(b) proportion.
Maintain function concept in application to problems.

The content of the above section is dealt with adequately in Book 3 with this exception. Trignometric ratios are presented very abruptly since these concepts were introduced in some detail in the previous text

in the series MATHEMATICS FOR CANADIANS — Book 2. Since this is the first occasion on which students are exposed to this topic, the teacher is advised to present a much more detailed development than is done in Book 3. Unfortunately, neither Aikens, pp. 334-347 or Welchons, pp. 328-341, are of much assistance, as they also assume previous knowledge of trignometric ratios and proceed quickly to more difficult aspects. MATHEMATICS FOR CANADIANS — Book 2, pp. 326-348, is a useful reference which is readily available in all senior high schools.

References:-

Book 3, pp. 50-77.
Aiken, pp. 334-347.
Welchons, pp. 328-341.

Special references for this section MATHEMATICS FOR CANADIANS — Book 2, pp. 326-348.

4. Factoring:-

(1) Factoring by the following methods:-

(a) common factor;
(b) trinomial;
(c) difference of squares;
(d) grouping;
(e) incomplete squares;
(f) sum and difference of cubes;
(g) Factor Theorem.

(2) Application of factoring:-

(a) H.C.F. and L.C.M.;
(b) multiplication and division of fractions;
(c) addition and subtraction of fractions;
(d) use of the multiplier (-1);
(e) complex fractions;

This topic is dealt with adequately in Book 3. The teacher may, however, wish to use one or another of the references for additional exercises.

References:-

Book 3, pp. 78-124.
Welchons, pp. 70-86, 100-124.
Dean/Moore, pp. 414-525.

5. Equations and Problems:-

(1) Literal equations — apply function concept.
(2) Fractional equations.
(3) Problems requiring the use of equations.

At this point the basic algebraic axioms listed previously in Section 1 should be reviewed. The basic principle spelled out on page 151, Book 3, should be emphasized, as it is vital to the solution of equations by factoring. Teachers with adequate knowledge of the postulates of the real number system, may wish to illustrate how this principle may be proven.

In teaching the solution of literal equations, emphasis should be placed upon the function notion: e.g. in $V = \frac{1}{2}H\ (a + b)$, V is a function of H, a and b;

in $H = \frac{2V}{a + b}$, H is a function of V, a, and b.

References:-

Book 3, pp. 144-173.
Aiken, pp. 24-74.
Welchons, pp. 100-124.
Dean/Moore, pp. 486-526.

6. Indices, Rational and Irrational Numbers:-

   A. Indices:-

      (1) Basic laws of indices.

      (2) Meaning of fractional, zero, and negative indices.

      (3) Application of rules of indices to: multiplication, division solving equations, factoring.

      (4) Logarithms:-

         (a) introduced as an application of indices;

         (b) use of logarithmic tables to facilitate:-

            i. multiplication;

            ii. division;

            iii. finding powers and roots.

   B. Rational and Irrational Numbers:-

      (1) Review the number system. Define rational number, irrational number.

      (2) Entire and mixed radicals.

      (3) Addition, subtraction, multiplication, and division of radicals.

      (4) Conjugate radicals.

      (5) Equations involving radicals.

The teacher should note that the order of the above topics varies in Book 3, Aiken and Welchons. This variation emphasizes that the teacher may use his own judgment regarding the order of presenting these concepts.

Regardless of order of presentation, the relationship between exponents and radicals as well as the relationship between exponents and logarithms should be stressed.

As was indicated in the discussion of topic 1 "Review of Basic Skills", the concept of rational, irrational, and real numbers could well be developed early in the course.

It should be noted that Aikens and Welchons stress that certain laws of exponents (properties of radicals) apply only if the variable does not equal zero. For example:-

$$\frac{a^{m}}{a^{n}} = a^{m-n}$$

if $a \neq 0$.

The teacher is advised to emphasize this point also.

The teacher of better classes, having taught logarithms, may wish to do as Welchons does, that is, teach the slide rule as an application of logarithms, thereby preparing students to use the slide rule in Grade XII chemistry and physics.

References:-

Book 3, pp. 125-143, 189-211, 292-323.
Aiken, pp. 75-135.
Welchons, pp. 202-228, 284-313.

7. Quadratic Equations:-

(1) Square root including the use of tables.

(2) Definitions for:-
(a) quadratic function;
(b) quadratic equation.

(3) Solution of the quadratic equation:-
(a) by factoring;
(b) by completing the square;
(c) by formula;
(d) graphically.

(4) The quadratic function:-
(a) significance of a, b, and c, in the general function $ax^2 + bx + c$, with regard to the sketch of the function;
(b) discussion of the graph of a quadratic function;
(c) correct use of language with reference to the graph of a quadratic function; maximum and minimum values, vertex, axis of symmetry, turning point, abscissa, ordinate.

(5) Problems leading to quadratic equations.

None of the references are satisfactory for topic 1. It is recommended that pp. 174-178 of Book 3 be taught followed by a lesson on the use of a table of squares and square roots. Book 3 is a satisfactory reference for topics 2, 3, and 5. For a complete discussion of topic 4, the teacher should consult either Aiken, pp. 274-287, or Welchons, pp. 246-257. In presenting this section, teachers should again stress the function notion and should emphasize the relation between linear and quadratic functions. As indicated earlier, graphing the circle, ellipse, and hyperbola may well be discussed in this section.

References:-

Book 3, pp. 174-178, 212-236.
Aiken, pp. 136-158, 219-245, 274-294.
Welchons, pp. 127-137, 246-257.

BIBLIOGRAPHY

Suggestions for the High School Mathematics shelf

The following are a few of the many titles recommended for the mathematics section of a high school library. Some are for the teacher as background and enrichment; many are suitable for alert and interested students. Approximate prices have been given for most publications.

(books starred (marked with *) are particularly recommended)

General Reference:-

Mathematics Dictionary Van Nostrand, Toronto	James & James $10.75 less 20%

History and Biography:-

Armitage, Angus	SUN, STAND THOU STILL. The life and work of Copernicus the Astronomer. New York, Schuman, 1947, 210 pages.
*Bell, E.T.	MEN OF MATHEMATICS. New York, Simon and Schuster, 1957, 592 pages. $6.50. A classic; contains piquant, terse characterizations of the personalities and achievements of some 35 outstanding mathematicians of all time.
Cahn, William	EINSTEIN: A PICTORIAL BIOGRAPHY. New York, Citadel Press, 1955, 128 pages (paper).
Cajori, Florian	A HISTORY OF ELEMENTARY MATHEMATICS. New York, Citadel Press, 1955, 128 pages (paper).
Freebury, H. A.	A HISTORY OF MATHEMATICS FOR SECONDARY SCHOOLS. London, Cassell, 1958.
Harsanyi, A. de.	THE STAR-GAZER. Trans. by Paul Tabor, New York, Putnam, 1939, 572 pages. Fictionalized biography of Galileo; intensely gripping.
Smith, D. E. and Ginsburg, J.	NUMBERS AND NUMERALS. Washington, D.C., National Council of Teachers of Mathematics, 1958, 52 pages, 35c.
Sullivan, J. W. M.	ISAAC NEWTON. 1642-1727.

Modern Mathematics:-

*Adler, Irving	THE NEW MATHEMATICS. New York, John Day, 1958, 187, pages, $3.75.
*Allendoerfer, C. B. and Oakley, C. O.	PRINCIPLES OF MATHEMATICS. New York, McGraw-Hill, 1955, 448 pages, $5.75.
Boehm, George A. W. and others	THE NEW WORLD OF MATHEMATICS. New York, Dial Press, 1959, 128 pages, $2.50 (paperback $1.50).
Dubisch	THE NATURE OF NUMBER; AN APPROACH TO BASIC IDEAS OF MODERN MATHEMATICS. Ronald Press, 1952, list price-$4.00.
	* 24th Yearbook of National Council of Teachers of Mathematics. The Growth of Mathematical Ideas, Grades K-12 (NCTM, 1959). Non-members $5.00.
*Evenson, A. B.	MODERN MATHEMATICS, INTRODUCTORY CONCEPTS AND THEIR IMPLICATIONS. Toronto, W. J. Gage Ltd., 1962, 206 pages, $2.00.

Johnson and Glenn — EXPLORING MATHEMATICS ON YOUR OWN Series. Toronto, Longmans Green. This is a fascinating series of enrichment booklets tailored for students who want to go beyond the textbook. Each booklet in the series presents a stimulating topic in an informal easy to read style. Each booklet is complete in itself and may be used by students for individual work or by the class as a whole. Ample practice is provided. $1.00 each.

*—Sets, Sentences, and Operations.
—The Pythagorean Theorem.
*—Invitation to Mathematics.
*—Understanding Numeration Systems.
—Fun with Mathematics.
—Number Patterns.
*—Topology—the Rubber-Sheet Geometry.
—The World of Statistics.
—Short Cuts in Computing.
*—The World of Measurement.
—Adventures in Graphing.
—All About Computing Devices.

*Kemeny, Snell, Thompson — INTRODUCTION TO FINITE MATHEMATICS. Englewood Cliffs, N.J., Prentice-Hall Inc., 1957, 372 pages, $8.25.

Krickenberger — SETS AND THE STRUCTURE OF ALGEBRA, Toronto, Ginn, pamphlet of 21 pages, 60c. A good introduction.

Ringenberg, L. A. — A PORTRAIT OF 2. Washington, D.C., National Council of Teachers of Mathematics, 1956, 42 pages (pamphlet), 75c.

Singh, Jagjit — MATHEMATICAL IDEAS. THEIR NATURE AND USE. London, Hutchinson, 1959, 312 pages.

Nature of Mathematics:-

*Bell, E. T. — MATHEMATICS: QUEEN AND SERVANT OF SCIENCE. New York, McGraw-Hill, 1951, 437 pages.

*Court, Nathan, Altshiller — MATHEMATICS IN FUN AND IN EARNEST. New York, Dial Press, 1958, 250 pages, $4.75.

*Kline, Morris — MATHEMATICS AND THE PHYSICAL WORLD. New York, Thomas Y. Crowell., Co. 1959, 482 pages, $6.50.

*Kline, Morris — MATHEMATICS IN WESTERN CULTURE. (Oxford, 1953), $6.00. This book gives a remarkably fine account of the influence mathematics has exerted on the development of philosophy, the physical sciences, religion, and the arts in Western life.

Lieber, Lillian R. and Hugh G. — THE EDUCATION OF T.C. MITS. (Norton, 1944), $3.95. A delightful, easy to read book that contains some interesting philosophy as well as mathematics.

Lieber, Lillian R. — With drawings by Hugh G. Lieber. NON-EUCLIDEAN GEOMETRY. (Galois Institute, 1940), $1.95. Various geometries needed for different surfaces, treated postulationally, making Euclidean and non-Euclidean geometries easier and interesting. Very readable.

*Logsdon, M. I. — A MATHEMATICIAN EXPLAINS. University of Chicago Press, 1936. Brief, very readable.

*Hogben, Lancelot — WONDERFUL WORLD OF MATHEMATICS. New York, Doubleday, $2.95. 1955.

Huff — HOW TO LIE WITH STATISTICS. (Norton, 1955), list price $1.95. Humorous, but penetrating and authoritative explanation of the basic conceptions and misconceptions of statistics. Valid illustrations in cartoon style fully capture and even extend the content.

Huff	HOW TO TAKE A CHANCE. (Norton, 1959), list price, $2.95. Entertaining but soundly exact discussions of various aspects of chance, probability, and error, especially as applied to everyday life.
Newman, James R.	THE WORLD OF MATHEMATICS. 4 volumes, New York, Simon and Schuster, 1956, $20.00. (Recommended for large schools). a veritable treasure house of every readable mathematical literature. Historical, biographical and expository, embracing mathematical development from ancient times to the present. Deals with the nature of mathematics, science and mathematics, mathematical thinking and invention, the cultural significance of mathematics and mathematical recreations. A unique anthology.
	NEW MATHEMATICS LIBRARY. A series of eight books prepared under the direction of the School Mathematics Study Group "in order to make some important mathematical ideas interesting and understandable to a large audience of high school students and laymen." Authors and titles are:- Beckenbach, Bellman—An Introduction to Inequalities. Davis—The Lore of Large Numbers. Kazarinoff—Geometric Inequalities. Niven—Numbers: Rational and Irrational. Salkind—The Contest Problem Book. Sawyer—What is Calculus About? Yaglom—Geometric Transformations. Zippin—Uses of Infinity. This series is published by the L. W. Singer Co. and is available from Random House of Canada Ltd., Toronto. $1.50 per book.
Sawyer, W. W.	PRELUDE TO MATHEMATICS. Baltimore, M.D. Penguin, 1955, 214 pages (paper). A stimulating popular exposition, including non-Euclidean geometry, matrix algebra, projective geometry, group theory.
Schaaf, W. L.	BASIC CONCEPTS OF ELEMENTARY MATHEMATICS. New York, John Wiley and Sons Inc., 1961, 386 pages, $6.00.
Recreational Mathematics:-	
Degrazia, Joseph	MATH IS FUN. New York, Gresham Press, 1948, 159 pages.
Gamow	ONE, TWO, THREE . . . INFINITY. (Mentor Books, 1947), list prict 40c. Problems of mathematics, physics, and astronomy clarified for the layman. This paperback should be available at the corner store.
Gardner, Martin	MATHEMATICS, MAGIC AND MYSTERY. New York, Dover, 1956, 176 pages. Emphasis on manipulative tricks and parlor magic involving mathematics.
Gardner, Martin	THE SCIENTIFIC AMERICAN BOOK OF MATHEMATICAL PUZZLES AND DIVERSIONS. New York, Simon and Schuster, 1959, 178 pages. A serious but lively presentation of significant aspects or mathematical recreation.
Jones, S. I.	MATHEMATICAL CLUBS AND RECREATIONS. Nashville, Jones, 1940, 256 pages. Indispensable for mathematics club programs and activities.
°Kraitchik	MATHEMATICAL RECREATIONS. (Dover, 1953), list price $1.75. One of the most thorough compilations of recreational mathematical problems. Highly recommended.
Mott-Smith, Geoffrey	MATHEMATICAL PUZZLES FOR BEGINNERS AND ENTHUSIASTS. New York, Dover, 1955, 248 pages, $1.00. Unusually fine collection of mathematical recreations, well presented.

ADDITIONAL BOOKS FOR THE TEACHERS' SHELF

18th Yearbook N.C.T.M. 1945. MULTI-SENSORY AIDS IN TEACHING MATHEMATICS. The most complete discussion available of teaching aids for mathematics. Fourth printing recently completed. 460 pages — $3.00.

21st Yearbook N.C.T.M. 1953. LEARNING OF MATHEMATICS, ITS THEORY AND PRACTICE. Applies the latest theories of the nature of the learning process to the problems of the classroom. 364 pages—$4.00. ($3.00 to members of the Council.)

*25th Yearbook N.C.T.M. 1960. INSTRUCTION IN ARITHMETIC. 374 pages, $3.50 and $4.50. Discusses value, nature and structure of arithmetic; factors affecting learning in arithmetic; modern mathematics and its impact on arithmetic; etc.

Butler and Wrenn: THE TEACHING OF SECONDARY MATHEMATICS. 1960 edition. McGraw-Hill. 550 pages, $7.20. This represents a complete revision of the earlier text. Implications of modern mathematics are carefully discussed and many suggestions are given for inclusion of appropriate materials within the framework of existing high school courses. The book should prove very helpful to Alberta teachers who are seeking to include aspects of modern mathematics in their current teaching.

Reeve, W. D.: MATHEMATICS FOR THE SECONDARY SCHOOLS. 1956. Holt, pages 547. Contains particularly good chapters on the teaching of geometry, both informal and demonstrative.

PERIODICALS

The Mathematics Teacher (8 times a year). Official journal of the National Council of Teachers of Mathematics, 1201 Sixteenth Street N. W., Washington, D.C. Teachers' membership including subscription, $5.25. Library rate $7.25 — a 'first' for teachers specializing in high school mathematics.

School Science and Mathematics (9 times a year, $4.50). P.O. Box 408, Oak Park, Illinois — official journal of Central Association of Science and Mathematics Teachers.

Scientific American (monthly). 415 Madison, New York 17, New York.

Mathematics Students Journal (quarterly — 5 subscriptions for $1.50, minimum order). N.C.T.M. —-1201 Sixteenth Street N.W., Washington, D.C. Good for high school mathematics clubs, enrichment classes, etc. Schools wishing to have more complete book list should obtain one or more of the following:

*1. **A Guide to the Use and Procurement of Teaching Aids for Mathematics** — Berger and Johnson, 75c.

2. **The High School Mathematics Library** — Wm. L. Shaaf — 40c. A very complete book list under 15 topics.
Both of the above are available from National Council of Teachers of Mathematics, 1201 Sixteenth Street N.W., Washington 6, D.C.

3. **Mathematics Book List for High School Libraries.** National High School and Junior College Mathematics Club, Mu Alpha Theta, Box 1155, University of Oklahoma, Norman, Oklahoma. (Send only self-addressed, stamped, No. 10 envelope.)

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